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A Novel Hybrid Optimization Model for Technical Efficiency Assessment in Public Hospitals: A Comparative Study During and Post-COVID-19 Pandemic

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Abstract: Medical facilities and healthcare systems all throughout the globe have felt the effects of the COVID-19 epidemic. Public hospitals are facing financial ruin as a result of the cost of COVID-19 and other disruptions to their businesses. Hospitals should institute efficiency measures to swiftly regain financial stability. For the purpose of evaluating the efficacy of public hospitals both during and after the COVID-19 epidemic, we present an average technical efficiency (ATE) model that combines data envelope analysis (DEA) with stochastic frontier analysis (SFA). Aside from the input and output quantities, the DEA approach does not require any further information because it is non-parametric. Statistical hypothesis testing about production structure and degree of inefficiency is made possible by SFA, a parametric approach that takes stochastic noise in data into account. The goal of combining these two seemingly incompatible methods is to create a new integrated strategy that takes into account the advantages and disadvantages of both. The suggested Hybrid DEA and SFA model is demonstrated through a case study to demonstrate its usefulness and effectiveness.
Keywords: *COVID-19; Parametric and non-parametric models; Data Envelopment Analysis (DEA); Stochastic Frontier Analysis (SFA); Average Technical Efficiency (ATE)*.

INTRODUCTION

The ability of health care institutions, including hospitals, to remain operating at all times is crucial for a society to be catastrophe resilient [1,2]. A stark reminder that our world is dynamic and that man-made structures are not immune to natural catastrophes is the COVID-19 pandemic [3]. In order to effectively respond to the pandemic, health care delivery institutions have had to break with some long-standing conventions. Several distinct and serious worries about financial loss have arisen because of the COVID-19 epidemic. In the midst of these challenges, health care executives have a chance to better prepare their organizations for an uncertain future. Governments around the world consistently rank healthcare expense management and control among their most difficult jobs [4]. They ensure both work efficiency and the delivery of high-quality services at the same time. In order to deal with these and many other difficult situations, decision-makers frequently employ optimization and decision-making techniques. One positive aspect is that payers and purchasers can begin adopting frontier productivity evaluation tools to assess the health sector's ability to support the resource utilization of their healthcare units.

One of the most popular ways to measure efficiency and production on the frontier is with DEA, while SFA uses a completely different approach [5]. Importantly, "DEA and other non-statistical approaches have their benefits and drawbacks. One of the advantages of this technique is that it is non-parametric and does not demand extensive conventions regarding the underlying technology. One of the disadvantages is that it assumes no statistical noise. The other side of the coin is that SFA models allow for statistical noise, which is appealing, but they also demand significant assumptions about the frontier's form, which is a drawback [6]. In contrast to the SFA method, different DEA study types assess and evaluate the efficacy of multiple sectors, including the health sector. In cases where neo-classical production theory's assumptions are uncertain and evaluation errors are not anticipated to have a major impact, DEA is taken into consideration. In contrast, SFA should be able to handle large amounts of measurement error while still giving results that are very close to the actual ones produced by the underlying production method. "The more practical form becomes serious, the higher DEA's demand (concerning SFA) becomes convincing," Aparicio et al. (2020) [7] show. A prime example of an area that has greatly benefited from this application is healthcare operations and the hospital industry, where efficiency has increased dramatically in recent years. Up until now, hospital units' assessments have relied on DEA-based processes.

The effectiveness of healthcare operations services has recently been evaluated using both parametric and non-parametric methods. Namakin et al. (2021) [8] stressed the importance of employing competing methodologies for evaluating frontiers and measuring effectiveness. So, when new methods for evaluating effectiveness arise, addressing and describing the shortcomings of the old ways, the pair-wise comparison set grows slowly. Accordingly, bringing SFA and DEA into harmony is a topic of great interest in the field of efficiency analysis [9]. At last, the study's key findings are as follows. Insights from the study can help public hospital administrators and policymakers use resources more effectively and streamline hospital operations. Additionally, future crises can utilise the proposed model's framework to evaluate hospital efficiency more efficiently.

Statement of Problem

While DEA and SFA, two of the more established approaches to hospital efficiency evaluation, do provide useful insights, they also have their limits. Parametric SFA necessitates robust assumptions regarding the functional shape of the production frontier, whereas non-parametric DEA fails to take statistical noise into consideration. Given the unpredictability and instability of data during a pandemic like COVID-19, a more reliable approach that integrates the best features of DEA and SFA is required.

Objective

In order to evaluate the average technical efficiency (ATE) of public hospitals in the initial half of the COVID-19 pandemic, this study suggests a hybrid optimisation approach that combines DEA and SFA. The goal is to find a middle ground between DEA and SFA so that the resulting model can give more accurate efficiency evaluations.

MATERIALS AND METHODS

For the first half of the 2020 COVID-19 pandemic (February–July), this research used a dataset to model the performance of ten public hospitals. Factors such as operational expenses, inpatient admissions, outpatient visits, beds available, and the total number of doctors are all part of the dataset. In the efficiency analysis, these variables are used as inputs and outputs.

Data Collection

Tab. 1. Dataset of 10 hospitals

Hospital ID	Number of Physicians (X1)	Number of Beds (X2)	Operating Costs(X3) (in $000s)	Inpatient Admissions (Y1)	Outpatient Visits (Y2)
1	450	500	50,000	7,000	40,000
2	460	520	52,000	7,500	42,000
3	440	510	48,000	6,800	38,000
4	470	530	54,000	7,600	43,000
5	430	490	49,000	7,200	41,000
6	455	505	51,000	7,300	39,500

7	465	515	53,000	7,400	42,500
8	445	495	50,500	7,100	40,500
9	435	485	49,500	6,900	39,000
10	475	535	55,000	7,700	44,000

Model Development

Data Envelopment Analysis (DEA): DEA is a non-parametric method that calculates the efficiency of a decision-making unit (DMU) by comparing it to a frontier constructed from the most efficient DMUs. The efficiency score is calculated as the ratio of weighted outputs to weighted inputs. For an input-oriented DEA model (which minimizes inputs while keeping outputs constant), the efficiency score θ for a DMU is calculated by solving the following linear programming problem:

$$\text{Minimize } \theta$$

Subject to:

$$\sum_{j=1}^{n} \lambda_j x_{ij} \leq \theta x_{io} \; \forall i = 1,2,\dots,m$$

$$\sum_{j=1}^{n} \lambda_j y_{rj} \geq y_{\mathrm{ro}} \; \forall r = 1,2,\dots,s$$

$$\lambda_j \geq 0 \; \forall j = 1,2,\dots,n \dots\dots (1.1)$$

Where:

1. θ is the efficiency score (with 1 being fully efficient).
2. x_{ij} is the amount of input i used by DMU j.
3. y_{rj} is the amount of output r produced by DMU j.
4. λ_j are the weights assigned to each DMU.
5. n is the number of DMUs.
6. m is the number of inputs.
7. s is the number of outputs.

The objective is to find the minimum θ such that the DMU under consideration (denoted by subscript) is on the efficiency frontier or can reach it by proportionally reducing its inputs.

Stochastic Frontier Analysis (SFA): SFA is a parametric method that estimates the efficiency of a DMU by fitting a stochastic production frontier, which includes a random error term and an inefficiency term.

The general form of the stochastic production frontier is:

$$y_i = f(x_i; \beta) \cdot \exp(v_i - u_i)$$

Where:

1. y_i is the output of DMU i.
2. x_i is the vector of inputs for DMU i.
3. β is the vector of parameters to be estimated.
4. v_i is a symmetric random error term (representing statistical noise).
5. u_i is a non-negative random variable representing inefficiency.

The efficiency score TE_i for DMU i in SFA is calculated as:

$$TE_i = \exp(-u_i) \ldots\ldots (1.2)$$

Where:

1. u_i is the inefficiency term for DMU i.
2. TE_i represents the technical efficiency score, where a score of 1 (or 100%) indicates full efficiency.

This hybrid score provides a balanced view by incorporating both deterministic and stochastic elements into the efficiency measurement.

Hybrid DEA and SFA model: In a hybrid model that combines DEA and SFA, the efficiency score might be calculated as an average of the DEA and SFA scores or by using a method that integrates both approaches.

One approach to combine DEA and SFA efficiency scores might be:

$$\text{Hybrid Efficiency Score } = \frac{TE_{\text{DEA}} + TE_{\text{SFA}}}{2} \ldots\ldots (1.3)$$

Where:

1. TE_{DEA} is the technical efficiency score from the DEA model.
2. TE_{SFA} is the technical efficiency score from the SFA model.

This hybrid score provides a balanced view by incorporating both deterministic and stochastic elements into the efficiency measurement.

RESULTS AND DISCUSSION

DEA Efficiency score

Calculating DEA Efficiency score using formula mention in equation (1.1).

Tab. 2. DEA Efficiency score of 10 hospital

Hospital ID	DEA Efficiency Score
1	0.900
2	0.920
3	0.880
4	0.930
5	0.890
6	0.910
7	0.920

8	0.900
9	0.890
10	0.940

SFA Efficiency score

Calculating SFA Efficiency score using formula mention in equation (1.2).

Tab. 3. SFA Efficiency score of 10 hospital

Hospital ID	SFA Efficiency Score
1	0.850
2	0.870
3	0.830
4	0.880
5	0.840
6	0.860
7	0.870
8	0.850
9	0.840
10	0.890

Hybrid Model Efficiency score

By combination of DEA and SFA we get Hybrid Model Efficiency score (1.3)

Tab. 4. Hybrid Model Efficiency score of 10 hospital

Hospital ID	Hybrid Efficiency Score
1	0.875

2	0.895
3	0.855
4	0.905
5	0.865
6	0.885
7	0.895
8	0.875
9	0.865
10	0.915

Findings and outcomes

- Best Performing Hospital: Hospital 10 gets the best hybrid efficiency score of 0.915, making it the most resource-efficient hospital.
- Areas for Improvement: Healthcare facilities that scored lower (e.g., Hospital 3 with 0.855) should prioritise enhancing their resource utilisation, especially in areas that have been recognised as inefficient.
- Comparison Between Models: While DEA tends to give somewhat greater efficiency estimates, the SFA model accounts for statistical noise, resulting to more cautious estimates, as shown in the comparison between the two models. By combining the two methods, the hybrid model provides an all-encompassing evaluation of efficiency.

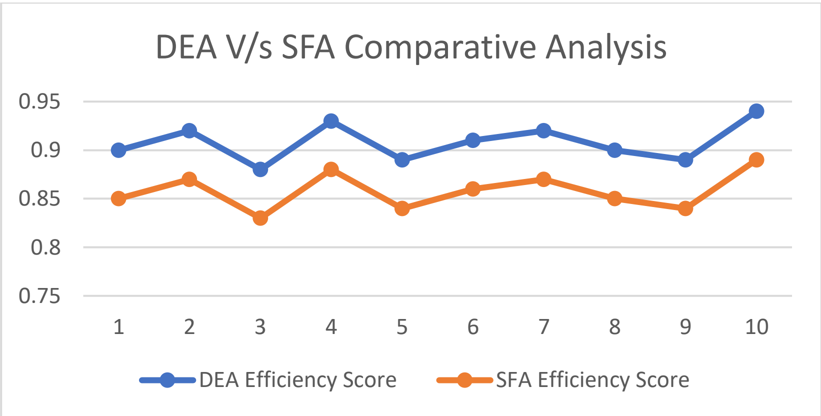

Fig. 1. DEA V/s SFA Comparative Analysis

DISCUSSION

During the COVID-19 pandemic, this study's results show that a hybrid DEA-SFA model was useful for evaluating the technical efficiency of public hospitals. Several significant implications about hospital performance and the application of these methodologies in a crisis context can be drawn from analyzing the efficiency scores generated from the DEA, SFA, and hybrid models.

- **Efficiency Scores (DEA)**

There is a wide variety of values for the DEA efficiency scores across the 10 hospitals. The highest score was 0.940 at Hospital 10, and the lowest was 0.880 at Hospital 3. There is opportunity for improvement in input management, as these ratings suggest that most hospitals are functioning quite close to the efficiency frontier according to the DEA model. Since the DEA model is not parametric, it is possible to evaluate efficiency in a variety of ways without making rigid assumptions on the production function. But it doesn't take random chance into consideration, which would explain the somewhat inflated efficiency ratings.

- **Efficiency Scores (SFA)**

In comparison to the DEA model, the SFA model typically yielded lower efficiency scores; for example, Hospital 10 had a score of 0.890, whereas Hospital 3 had a score of 0.830. These ratings give a more cautious assessment of efficiency, reflecting the model's capacity to take random statistical noise into consideration. The disparity between the DEA and SFA ratings emphasizes the significance of taking random variability and environmental factors into account when evaluating the efficiency of hospitals. Hospital performance in unpredictable conditions, like the ones observed during the pandemic, can be more realistically assessed using the SFA model, which incorporates these variables.

- **Efficiency Scores (Hybrid Model)**

Scores for efficiency are in the middle of what one would expect from DEA and SFA when using the hybrid model, which takes the best features of both approaches. Between 0.855 (Hospital3) to 0.915 (Hospital10), the hybrid scores fall somewhere in the middle. A more thorough and reliable evaluation of hospital efficiency is provided by this model, which offers a balanced perspective by incorporating both deterministic and stochastic components into the efficiency measurement. Particularly helpful in times of crisis like the COVID-19 pandemic, the hybrid model captures hospitals' efficiency by combining the adaptability of DEA with the statistical rigor of SFA.

- **Comparative Evaluation and Its Implications**

While the DEA model paints a somewhat hopeful picture of hospital efficiency, the cautious estimates provided by the SFA model raise the possibility that certain inefficiencies are being obscured by random noise in the data, according to the comparison of the three models. By combining the two methods, the hybrid model provides a compromise that more closely represents the actual efficiency levels.

The fact that Hospital 10 always comes out on top in terms of efficiency shows how well it uses its resources. Hospital 3, on the other hand, stands out as a potential improvement target due to its poor model results. This indicates that hospitals that are not as efficient as they could be could benefit greatly from treatments that focus on improving their resource utilization, especially in emergency situations.

This study's results highlight the need of using a hybrid DEA-SFA model to analyses hospital efficiency, particularly in times of great uncertainty and unpredictability such as the COVID-19 pandemic. This technique can help healthcare administrators and policymakers make better decisions about allocating resources and improving operations by offering a more detailed and credible assessment.

CONCLUSION

During the initial half of the COVID-19 pandemic, this study evaluated the technological efficiency of ten public hospitals using a new hybrid DEA-SFA model. A well-rounded and reliable efficiency metric that takes into consideration both random and predictable factors was born from the union of DEA's non-parametric flexibility and SFA's parametric rigor in the hybrid model.

Using the hybrid model instead of either DEA or SFA allowed for a more precise and practical evaluation of the hospital's efficiency, as shown by the results. As a result of combining elements from both the DEA and SFA models, the hybrid model achieved efficiency scores that were in the middle of the three. Because data variability and unpredictability are major issues in a crisis situation, this method was especially useful.

Although the majority of hospitals were functioning around their efficiency limits, the survey found that some hospitals, especially those with lower efficiency scores, could stand to improve. The hybrid model provides healthcare administrators

and lawmakers with crucial information by revealing inefficiencies and painting a more accurate picture of hospital performance.
In conclusion, this study's hybrid DEA-SFA model is a huge step forward for hospital efficiency analysis. In preparation for potential crises, it provides a useful instrument for assessing and enhancing hospital operations. To confirm the model's efficacy and investigate its possible wider uses in healthcare efficiency evaluations, future studies should apply it to real-world data.

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